Advanced Quantum Theory **WS 2015/16**

Problem Set 3 Due: 5/6 November 2015

Problem 7 Harmonic Oscillator (Oral)

The Hamiltonian for the harmonic oscillator is given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2, \tag{1}$$

(a) Find the time dependence of the expectation values of the initial position and initial momentum operators:

$$x_0 = x\ cos(\omega t) - (p/m\omega)\ sin(\omega t)$$
$$p_0 = p\ cos(\omega t) + m\omega x\ sin(\omega t)$$

(b) Do these operators commute with the Hamiltonian?

(c) Do you find your results for (a) and (b) to be compatible?

(d) What are the motion equations of the operators in the Heisenberg picture?

(e) Compute the commutator $[p_0{,}x_0]$. What is its significance for measurement theory?

Problem 8 Rabi Oscillations (Oral)

The Hamiltonian for a two-state system is given by

$$\hat{H} = \frac{\hbar\Omega}{2}\left(|+\rangle\langle-| + |-\rangle\langle+|\right). \tag{2}$$

Define the Schrödinger picture operators

$$\hat{\sigma}_+ = |+\rangle\langle-| \qquad \hat{\sigma}_- = |-\rangle\langle+| \qquad \hat{\sigma}_z = |+\rangle\langle+| - |-\rangle\langle-|\,. \tag{3}$$

(a) Calculate the commutation relations of $\hat{\sigma}_{\pm,z}$ with each other and with $\hat{H}$.

(b) Write down the Heisenberg equations of motion for $\hat{\sigma}_{\pm,z}(t)$ and solve them.

Check: $\hat{\sigma}_z(t) = i(\hat{\sigma}_- - \hat{\sigma}_+)\sin\Omega t + \hat{\sigma}_z\cos\Omega t$

(c) The Heisenberg state vector is $|\psi\rangle_H = |+\rangle$. Working in the Heisenberg picture, find the probability that the system is found to be in the state $|\pm\rangle$ at time t.

Hint: Express $|+\rangle\langle+|$ *in terms of* $\hat{\sigma}_{\pm,z}$.

Problem 9 Commutation Relation (Written)

Using the coordinate-momentum commutation relation prove that:

$$\sum_n (E_n - E_0)|\langle n|x|0\rangle|^2 = constant, \tag{4}$$

where E_n is the energy corresponding to the eigenstate $|n\rangle$. Obtain the value of the constant. The Hamiltonian has the form $H = \frac{p^2}{2M} + V(x)$