**Advanced Quantum Theory** **WS 2015/16**
**Problem Set 4** Due: 12/13 November 2015

### Problem 10 Coherent States (Oral)

A coherent state of a one dimensional simple harmonic oscillator is defined to be an eigenstate of the non-Hermitian annihilation operator $\hat{a}$:

$$\hat{a}\,|\lambda\rangle = \lambda\,|\lambda\rangle \tag{1}$$

Where $\lambda$ is, in general a complex number.

(a) Prove that

$$|\lambda\rangle = e^{-|\lambda|^2/2} e^{\lambda a^\dagger}\,|0\rangle \tag{2}$$

is a normalized coherent state.

(b) Write $|\lambda\rangle$ as

$$\lambda = \sum_{n=0}^{\infty} f(n)\,|n\rangle \tag{3}$$

Show that the distribution of $|f(n)|^2$ with respect to $n$ is of the Poissoin form. Find the most probable value of $n$, hence of $E$.

(c) Show that a coherent state can also b e obtained by applying the translation (finite displacement) operator $e^{ipl/\hbar}$ (where $p$ is the momentum operator and $l$ is the displacement ) to the ground state.

(d) Show that the coherent state $\lambda$ remains coherent under time evolution and calculate the time-evolved state $|\lambda(t)\rangle$ (Hint: directly apply the time-evolution operator.)

### Problem 11 Correlation Function (Oral)

Consider a function, known as **correlation function** defined by

$$C(t) = \langle x(t)x(0)\rangle \tag{4}$$

where $x(t)$ is the position operator in the Heisenberg picture. Evaluate the correlation function explicitly for the ground state of a one dimensional simple harmonic oscillator.

### Problem 12 Translation operator (Written)

Consider the translation operator defined by

$$\hat{T}(a)\,|x\rangle = |x+a\rangle\,, \tag{5}$$

where $|x\rangle$ is the coordinate basis and $a$ a real constant.

(a) Convince yourself that when $\hat{T}(a)$ acts on a wave function we obtain

$$\hat{T}(a)\psi(x) = \psi(x-a). \tag{6}$$

(b) Show that the translation operator commutes with the momentum operator.

(c) Show that $\hat{T}(a)$ is a unitary operator, $\hat{T}^{\dagger}(a)\hat{T}(a) = \hat{1}$. What are its eigenvalues and associated eigenvectors? Further show that it is sufficient to know the wave function within an interval of length $a$ only.