Problem Set 5 Due: 19/20 November 2015

Problem 14 Perturbed Harmonic Oscillator (Oral)

A harmonic oscillator is subject to the perturbation

$$\hat{V}(t) = \frac{\sqrt{3}}{2}\hbar\omega\Theta(t)(|2\rangle\langle 1| + |1\rangle\langle 2|), \tag{1}$$

where $\Theta(t)$ is the Heaviside step function. Given that the oscillator is prepared in the state $|n = 2\rangle$ for $t \leq 0$, compute the probability amplitudes of the various unperturbed oscillator states $\langle n|\psi(t)\rangle$ at time $t > 0$,

(a) by solving the probability amplitude differential equations directly

(*Hint: It is useful to first scale the differential equations by defining a dimensionless time* $\tau = \omega t/2$.)

(b) by computing the eigenvectors and eigenvalues of the Hamiltonian $\hat{H}_0 + \hat{V}$.

(c) Make a sketch of the lowest few energy eigenvalues of the system Hamiltonian for $t < 0$ and $t > 0$, side by side.

Problem 15 Rotational Motion of a Rigid Body (Written)

Consider a diatomic molecule composed of two identical atoms rotating about its center of mass in a space-fixed coordinate system. For simplicity assume that the distance between the atoms is fixed.[1] The Hamiltonian of the molecule is:

$$\hat{H} = \frac{\hat{\mathbf{L}}^2}{2I}, \tag{2}$$

where I is the moment of inertia of the molecule with respect to the rotational axis.

(a) What are the energy eigenvalues and eigenfunctions of the molecule and what are the degeneracies of the energies? Determine the energy difference between two rotational levels l and $l + 1$.

(b) Assume the molecule is prepared in the state

$$\psi(\theta, \phi) = A(\cos^2\theta + \sin^2\theta\cos 2\phi), \tag{3}$$

where A is a normalization constant.

(i) What are the probabilities of measuring the values $6\hbar^2$, $2\hbar^2$ and $0\hbar^2$ for $\hat{\mathbf{L}}^2$?

(ii) What is the probability of measuring simultaneously the pair $(6\hbar^2, -2\hbar)$ for $\hat{\mathbf{L}}^2$ and $\hat{L}_z$?

Hint: Express $\psi(\theta, \phi)$ *in terms of the spherical harmonics* $Y_{lm}(\theta, \phi)$ *and determine* A.

[1] In the literature such a system is also called a **rigid rotor**.

Problem 16 Spin-$\frac{1}{2}$ in a Magnetic Field (Oral)

A particle with spin 1/2 and magnetic moment μ is placed in a magnetic field

$$\mathbf{B} = B_0\hat{\mathbf{z}} + B_1 cos(wt)\hat{\mathbf{x}} - B_1 sin(wt)\hat{\mathbf{y}} \tag{4}$$

which is often employed in magnetic resonance experiments.

Assume that the particle has spin up along $+z$ -axis at $t = 0$ ($m_s = +1/2$). Derive the probability to find the particle with spin down ($m_s = -1/2$) at time $t > 0$