Advanced Quantum Theory **WS 2015/16**

Problem Set 6 Due: 26/27 November 2015

Problem 17 Angular momentum (Written)

An operator f describing the interaction of two spin 1/2 particles has the form

$$f = a + b\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2} \tag{1}$$

where a and b are constants and σ_1 and σ_2 are Pauli matrices. The total spin angular momentum is

$$\mathbf{J} = \mathbf{j_1} + \mathbf{j_2} = \hbar/\mathbf{2}(\boldsymbol{\sigma_1} + \boldsymbol{\sigma_2})$$

(a) Show that f, $\mathbf{J^2}$ and J_z can be simultaneously measured.

(b) Derive the matrix representation for f in the $|J, M, j_1, j_2|$ basis. (Label rows and columns of matrix)

(c) Derive the matrix representation for f in the $|j_1, j_2, m_1, m_2|$ basis.

Problem 18 Expectation value of $\hat{L}_x$ (Oral)

A system is prepared in a state of angular momentum given by

$$\Psi = aY_{1,1} + bY_{1,0} + cY_{1,-1}, \tag{2}$$

where $|a|^2 + |b|^2 + |c|^2 = 1$ and $Y_{l,m}$ denote the spherical harmonics.

(a) Calculate the expectation value of $\hat{L}_x$.

(b) Calculate the expectation value of $\hat{\boldsymbol{L}}^2$.

(c) Determine the coefficients a, b, c such that $\hat{L}_x\Psi = \hbar\Psi$.

Hint:

$$\hat{L}_\pm Y_{l,m} = \sqrt{l(l+1) - m(m \pm 1)}\hbar Y_{l,m\pm1} \quad \text{and} \quad \hat{L}_x = \frac{1}{2}(\hat{L}_+ + \hat{L}_-)$$

Problem 19 Magnetic Resonance (Oral)

Consider a spin-$\frac{1}{2}$ particle subject to the rotating magnetic field

$$\boldsymbol{B}(t) = B_1(\cos\omega t\boldsymbol{e}_x + \sin\omega t\boldsymbol{e}_y) + B_0\boldsymbol{e}_z. \tag{3}$$

The system Hamiltonian is given by

$$\hat{H} = -\mu\hat{\boldsymbol{S}} \cdot \boldsymbol{B}(t), \tag{4}$$

where $\hat{\boldsymbol{S}} = \frac{\hbar}{2}\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$.

(a) Rewrite the Hamiltonian as a 2×2 matrix in the $\{|s, m_s\rangle\} \equiv \{|+\rangle, |-\rangle\}$ basis. Using the time-dependent Schrödinger equation and $|\psi(t)\rangle = a_+ |+\rangle + a_- |-\rangle$, determine the differential equations for the amplitudes a_+ and a_- and solve them with an appropriate ansatz.

(b) If the particle is at $t = 0$ in the state $|+\rangle$, what is the probability of finding the particle in the state $|-\rangle$ at time t? Under what condition will the probability be highest?