
Problem Set 8 Due: 10/11 December 2015

Problem 24 Perturbation Theory (Oral)

Consider a two dimensional harmonic oscillator

$$H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) \tag{1}$$

The Hamiltonian is given in units of $\hbar = m == \omega = 1$

(a) What are the wave functions and energies of the 3 lowest states?

(b) Next consider a perturbation to the Hamiltonian

$$V = \frac{1}{2}\epsilon xy(x^2 + y^2), \quad (\epsilon \ll 1) \tag{2}$$

Compute to first order in perturbation theory the effect of V on the energies of the states calculated in part (a).

Hint: The wave function and the energy of 2D H.O are

$$\psi_{n_1,n_2} = N_{n_1,n_2} e^{-(x^2+y^2)/2} H_{n_1}(x) H_{n_2}(y), \tag{3}$$

$$E_{n_1,n_2} = n_1 + n_2 + 1 \tag{4}$$

Where H_i are Hermite polynomials.

Problem 25 3-Level System (Oral)

A system that has three unperturbed states can be represented by the perturbed Hamiltonian matrix

$$\hat{H} = \begin{pmatrix} E_1 & 0 & a \\ 0 & E_1 & b \\ a^* & b^* & E_2 \end{pmatrix}, \tag{5}$$

where $E_2 > E_1$. The quantities a and b are to be regarded as perturbations that are of the same order and small compared to $E_2 - E_1$.

(a) Use second order non-degenerate perturbation theory to calculate the perturbed eigenvalues. Is this procedure correct?

(b) Diagonalize the matrix to find the exact eigenvalues.

(c) Use second order degenerate perturbation theory. Compare the results obtained.

Problem 26 Interacting spins (Written)

A system of 3 (non identicle) spin one half particles, whose spin operators are $\vec{S_1}$, $\vec{S_2}$, and $\vec{S_3}$, is governed by the Hamiltonian

$$H = A\frac{\vec{S_1}.\vec{S_2}}{\hbar^2} + B\frac{(\vec{S_1}+\vec{S_2}).\vec{S_3}}{\hbar^2} \tag{6}$$

Find the energy levels and their degeneracies.

Problem 27 spin 1/2 particle in magnetic field (Oral)

The Hamiltonian for a spin 1/2 particle with charge $+e$ in an external magnetic field is,

$$H = -\frac{ge}{2mc}\vec{S}.\vec{B} \tag{7}$$

Calculate the operator $\frac{d\vec{s}}{dt}$ if $\vec{B} = B\hat{y}$. What is $S_z(t)$ in matrix form?