Problem Set 9 Due: 17/18 December 2015

Problem 26 Perturbation (Written)

Consider the one dimensional motion of an electron confined to a potential well $V(x) = \frac{1}{2}kx^2$ and subjected also to a perturbing electric field $\vec{F} = F\hat{x}$.

(a) Determine the shift in the energy levels of this system due to the electric field.

(b) The dipole moment of this system in state n is defined as $P_n = -e < x >_n$, where $< x >_n$ is the expectation value of x in the state n. Find the dipole moment of the system in the presence of the electric field.

Problem 27 Rabi Oscillations (Oral)

A two-state system has eigenstates $|1\rangle$ and $|2\rangle$ with energy levels E_1 and E_2, respectively, and $E_2 > E_1$. A time-dependent perturbation connects the two levels as follows:

$$\hat{V}(t) = |1\rangle \langle 2| V_0 e^{i\omega t} + |2\rangle \langle 1| V_0 e^{-i\omega t}. \tag{1}$$

(a) Write the state vector as a linear combination of the unperturbed energy eigenstates

$$|\psi(t)\rangle = \sum_k c_k(t) e^{-iE_k t/\hbar} |k\rangle . \tag{2}$$

Deduce the coupled differential equations for $c_k(t)$ and solve them exactly for the case $c_1(0) = 1$ and $c_2(0) = 0$. Write down explicitly the values of $|c_k(t)|^2$ for $k = 1, 2$.

(b) Do the same problem using time-dependent perturbation theory to lowest non-vanishing order. Compare your results with (a) for small V_0. Treat the following two cases separately:

(i) ω very different from $\frac{1}{\hbar}(E_2 - E_1)$ and
(ii) ω very close to $\frac{1}{\hbar}(E_2 - E_1)$.

Problem 28 Deuteron (Oral)

Assume that a deuteron is an $l = 0$ bound state of a neutron and a proton in the potential

$$V(r) = -V_0 e^{-r/a}, \tag{3}$$

where $V_0 = 32$ MeV and $a = 2.2$ fm. Using the trial function

$$\psi(\alpha) = Ae^{-\alpha r/a} \tag{4}$$

and the variational method determine the binding energy of the deuteron.