1 a) $\frac{d}{dy}(ay+b)(cy+d)^{-1} = \frac{a}{cy+d} - \frac{c(ay+b)}{(cy+d)^2}$ $\frac{d}{dy}\frac{ay+b}{cy+d} = \frac{a(cy+d)-c(ay+b)}{(cy+d)^2} = \frac{ad-bc}{(cy+d)^2}$

b) $\frac{d}{dz} e^{\beta z^2}\cdot\cos(1-3z) = 2\beta z\, e^{\beta z^2}\cos(1-3z) + 3\sin(1-3z)\, e^{\beta z^2}$

c) $\frac{d}{dx}(1+(1+x^{1/2})^{1/2})^{1/2} = \frac{1}{2}(1+(1+x^{1/2})^{1/2})^{-1/2}\cdot\frac{1}{2}(1+x^{1/2})^{-1/2}\cdot\frac{1}{2}x^{-1/2} =$

$= \frac{1}{8}\frac{1}{\sqrt{1+\sqrt{1+\sqrt{x}}}\cdot\sqrt{1+\sqrt{x}}\cdot\sqrt{x}}$

d) $\frac{d}{dk}\cos(\sin(\cos(k))) = -\sin(\sin(\cos(k))\cdot\cos(\cos(k))\cdot(-\sin(k))$

2 a) $\int\frac{e^{ax}}{1+e^{ax}}dx$ $y = e^{ax}$ $\frac{dy}{dx} = ae^{ax} = ay$

$= \frac{1}{a}\int\frac{1}{1+y}dy = \frac{1}{a}\ln(|1+y|)+C = \frac{1}{a}\ln((1+e^{ax}))+C$

b) $\int\sin^3 u\cos^3 u\,du$ $x=\sin(u)$ $\frac{dx}{du}=\cos(u)$

$\int x^3(1-x^2)dx = \frac{1}{4}x^4 - \frac{1}{6}x^6 + C$

$= \frac{1}{4}\sin^4(u) - \frac{1}{6}\sin^6(u) + C$

Alternativ

$\int\sin^3 u\cos^3 u\,du$ $x=\cos(u)$ $\frac{dx}{du} = -\sin(u)$

$= \int x^3(1-x^2)dx = -\frac{1}{4}x^4 + \frac{1}{6}x^6 + C$

$= -\frac{1}{4}\cos^4(u) + \frac{1}{6}\cos^6(u) + C$

2b) Alternativ

$$\int \sin^3(u)\cos^3(u)\,du$$

$$= \frac{1}{8}\int \sin^3(2u)\,du = \frac{1}{8}\int \underbrace{\sin(2u)}_{u'}\,\underbrace{\sin^2(2u)}_{v}\,du$$

$$= \frac{1}{8}\left(-\frac{1}{2}\cos(2u)\sin^2(2u) + \frac{1}{2}\int \cos(2u)\,2\sin(2u)\cdot\cos(2u)\cdot 2\,du\right)$$

$$= \frac{1}{8}\left(-\frac{1}{2}\cos(2u)\sin^2(2u) + 2\underbrace{\int \sin(2u)\cos^2(2u)\,du}_{I}\right)$$

$$x = \cos(2u) \Rightarrow \frac{dx}{du} = -2\sin(2u)$$

$$I = -\frac{1}{2}\int x^2\,dx = -\frac{1}{6}x^3 + C$$

$$= -\frac{1}{6}\cos^3(2u) + C$$

$$= \frac{1}{8}\left(-\frac{1}{2}\cos(2u)\sin^2(2u) - \frac{1}{3}\cos^3(2u)\right) + C$$

2c)

$$3\int_0^{\ln(2)} \underbrace{x^2}_{v}\, \underbrace{e^{-x}}_{u'}\,dx = \underbrace{3\left[-e^{-x}\cdot x^2\right]_0^{\ln(2)}}_{-3\left(\frac{1}{2}\ln^2(2)\right)} + \underbrace{3\int_0^{\ln(2)} e^{-x}\cdot x\cdot 2\,dx}_{+\,3I}$$

$$I = 2\int_0^{\ln(2)} \underbrace{e^{-x}}_{u'}\,\underbrace{x}_{v}\,dx = 2\left[-e^{-x}\cdot x\right]_0^{\ln(2)} + 2\int_0^{\ln(2)} e^{-x}\,dx$$

$$= 2\left(-\frac{1}{2}\ln(2) - \left[e^{-x}\right]_0^{\ln(2)}\right)$$

$$= 2\left(-\frac{1}{2}\ln(2) - \left(\frac{1}{2}-1\right)\right) = 2\left(\frac{1}{2} - \frac{1}{2}\ln(2)\right)$$

$$= \frac{3}{2}\left(2 - \overbrace{2\ln(2)}^{\ln(4)} - \ln^2(2)\right)$$

2d

$$\int_1^e \underbrace{x^2}_{u'}\,\underbrace{\ln^2(x)}_{v}\,dx = \left[\frac{1}{3}x^3\ln^2(x)\right]_1^e - \frac{1}{3}\int_1^e x^3\cdot 2\ln x\cdot\frac{1}{x}\,dx$$

$$= \frac{1}{3}e^3 - \frac{2}{3}\int_1^e \underbrace{x^2}_{u'}\,\underbrace{\ln x}_{v}\,dx$$

$$= \frac{1}{3}e^3 - \frac{2}{3}\left(\left[\frac{1}{3}x^3\ln x\right]_1^e - \frac{1}{3}\int_1^e x^3\frac{1}{x}\,dx\right)$$

$$= \frac{1}{3}e^3 - \frac{2}{3}\left(\frac{1}{3}e^3 - \frac{1}{9}\left[x^3\right]_1^e\right)$$

$$= \frac{1}{3}e^3 - \frac{2}{9}e^3 + \frac{2}{27}\left(e^3-1\right) = \frac{5}{27}e^3 - \frac{2}{27}$$

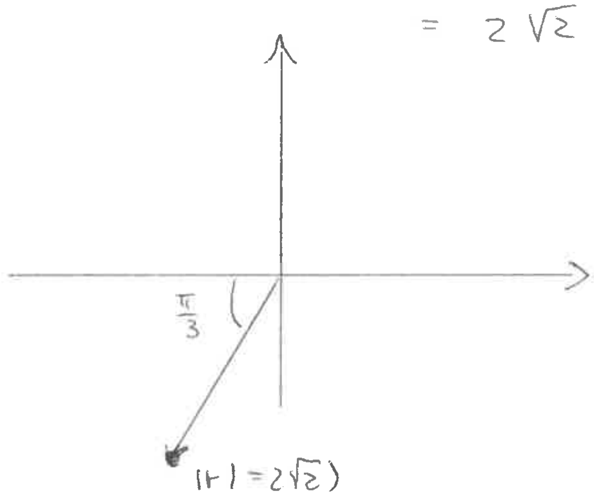
5.) $z = -\left(\frac{4\sqrt{2}}{1+i\sqrt{3}}\right)^{*}$

$\frac{4\sqrt{2}(1-i\sqrt{3})}{1+\sqrt{3}^2} = \sqrt{2} - i\sqrt{2}\sqrt{3}$

$\Rightarrow z = -\sqrt{2} - i\sqrt{2}\sqrt{3} = -2\sqrt{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -2\sqrt{2}\; e^{i\pi/3}$

$= 2\sqrt{2}\; e^{-i\frac{2\pi}{3}}$



$-\overline{\left(\frac{4\sqrt{2}}{1+i\sqrt{3}}\right)\left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)} = -\overline{\left(\frac{4\sqrt{2}(1-i\sqrt{3})}{1+\sqrt{3}^2}\right)} = -\left(\sqrt{2} + i\sqrt{2}\sqrt{3}\right)$

3.) $X^T = \begin{pmatrix} -3 & 0 & 2 & 4 \\ 0 & 2 & 0 & 3 \\ 1 & 0 & -1 & 1 \end{pmatrix}$

$X^T \cdot Y = \begin{pmatrix} -3 & 0 & 2 & 4 \\ 0 & 2 & 0 & 3 \\ 1 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -2 & 1 \\ 3 & 0 & 0 \\ 0 & 1 & 3 \\ 1 & -4 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -8 & -1 \\ 9 & -12 & -3 \\ 1 & -7 & -3 \end{pmatrix}$

In[11]:= Int(e^(a*x)/(1+e^(a*x))) dx

Indefinite integrals:

Approximate form | Hide steps

$$\int \frac{e^{a x}}{1+e^{a x}}\, dx = \frac{\log(e^{a x}+1)}{a} + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int \frac{e^{a x}}{e^{a x}+1}\, dx$$

For the integrand $\frac{e^{a x}}{e^{a x}+1}$, substitute $u = a x$ and $du = a\, dx$:

$$= \frac{1}{a} \int \frac{e^{u}}{e^{u}+1}\, du$$

For the integrand $\frac{e^{u}}{e^{u}+1}$, substitute $s = e^{u}+1$ and $ds = e^{u}\, du$:

$$= \frac{1}{a} \int \frac{1}{s}\, ds$$

The integral of $\frac{1}{s}$ is $\log(s)$:

$$= \frac{\log(s)}{a} + \text{constant}$$

Substitute back for $s = e^{u}+1$:

$$= \frac{\log(e^{u}+1)}{a} + \text{constant}$$

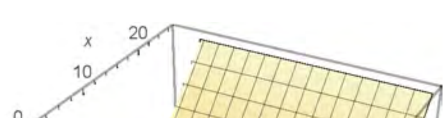
Substitute back for $u = a x$:

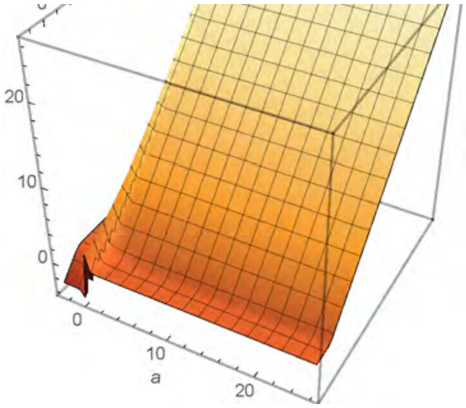
Answer:

$$= \frac{\log(e^{a x}+1)}{a} + \text{constant}$$

log(x) is the natural logarithm »

3D plot:



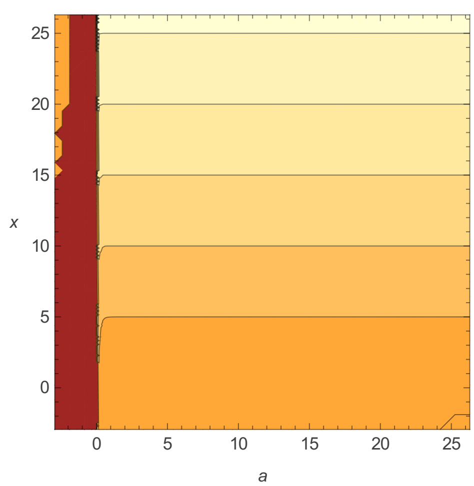


a_{min} a_{max}

x_{min} x_{max}

+ More controls

Contour plot:



a_{min} a_{max}

x_{min} x_{max}

Series expansion of the integral at x=0:

$$\frac{\log(2)}{a} + \frac{x}{2} + \frac{a\,x^2}{8} - \frac{a^3\,x^4}{192} + O(x^6)$$

(Taylor series)

Big- O notation »

In[5]:= Int(3*x^2*exp(-x)) dx from 0 to ln(2)

Definite integrals:

More digits Hide steps

$$\int_0^{\log(2)} 3\, x^2 \exp(-x)\, dx = -\frac{3}{2}\left(-2 + \log^2(2) + \log(4)\right) \approx 0.19988$$

Possible intermediate steps:

Compute the definite integral:

$$\int_0^{\log(2)} 3\, e^{-x}\, x^2\, dx$$

Factor out constants:

$$= 3 \int_0^{\log(2)} e^{-x}\, x^2\, dx$$

For the integrand $e^{-x}\, x^2$, integrate by parts, $\int f\, dg = f\, g - \int g\, df$, where

$f = x^2, \quad dg = e^{-x}\, dx,$
$df = 2\, x\, dx, \quad g = -e^{-x}$:

$$= \left(-3\, e^{-x}\, x^2\right)\Big|_0^{\log(2)} + 6 \int_0^{\log(2)} e^{-x}\, x\, dx$$

Evaluate the antiderivative at the limits and subtract.

$\left(-3\, e^{-x}\, x^2\right)\Big|_0^{\log(2)} = \left(-3\, e^{-\log(2)} \log^2(2)\right) - \left(-3\, e^{-0}\, 0^2\right) = -\frac{3}{2} \log^2(2)$:

$$= -\frac{3}{2} \log^2(2) + 6 \int_0^{\log(2)} e^{-x}\, x\, dx$$

For the integrand $e^{-x}\, x$, integrate by parts, $\int f\, dg = f\, g - \int g\, df$, where

$f = x, \quad dg = e^{-x}\, dx,$
$df = dx, \quad g = -e^{-x}$:

$$= -\frac{3}{2} \log^2(2) + \left(-6\, e^{-x}\, x\right)\Big|_0^{\log(2)} + 6 \int_0^{\log(2)} e^{-x}\, dx$$

Evaluate the antiderivative at the limits and subtract.

$\left(-6\, e^{-x}\, x\right)\Big|_0^{\log(2)} = \left(-6\, e^{-\log(2)} \log(2)\right) - \left(-6\, e^{-0}\, 0\right) = -\log(8)$:

$$= -\frac{3}{2} \log^2(2) - \log(8) + 6 \int_0^{\log(2)} e^{-x}\, dx$$

Apply the fundamental theorem of calculus.
The antiderivative of e^{-x} is $-e^{-x}$:

$$= -\frac{3}{2}\log^2(2) - \log(8) + \left(-6\,e^{-x}\right)\Big|_0^{\log(2)}$$

Evaluate the antiderivative at the limits and subtract.

$\left(-6\,e^{-x}\right)\Big|_0^{\log(2)} = \left(-6\,e^{-\log(2)}\right) - \left(-6\,e^{-0}\right) = 3$:

$$= 3 - \frac{3\log^2(2)}{2} - \log(8)$$

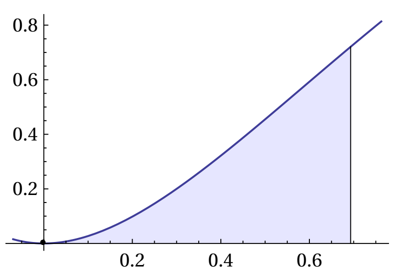
Which is equal to:

Answer:

$$= -\frac{3}{2}\left(-2 + \log^2(2) + \log(4)\right)$$

log(*x*) is the natural logarithm »

Visual representation of the integral:

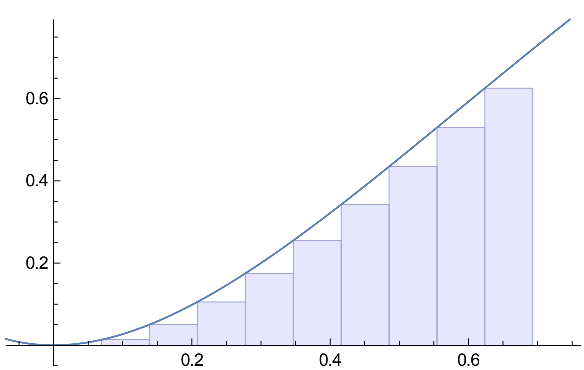


Riemann sums:

More cases

left sum	$-\frac{1}{\left(2^{\frac{1}{n}}-1\right)^3 n^3} 3 \times 2^{\frac{1}{n}-1}\left(\left(2^{\frac{1}{n}}-1\right)^2 n^2 + 2\left(2^{\frac{1}{n}}-1\right)n - 2^{\frac{1}{n}} - 1\right)\log^3(2) = \left(3 - \frac{3\log^2(2)}{2} - 3\log(2)\right) - \frac{3\log^3(2)}{4n} + O\left(\left(\frac{1}{n}\right)^2\right)$

(assuming subintervals of equal length)



integral: $-\frac{3}{2}(-2+\log^2(2)+\log(4)) \approx 0.199879$

Riemann sum: 0.175446

error: 0.0244333

number of subintervals

summation method: left endpoint | midpoint | right endpoint

Indefinite integral:

Step-by-step solution

$$\int 3\,x^2 \exp(-x)\,dx = 3\,e^{-x}\,(-x^2 - 2\,x - 2) + \text{constant}$$

WolframAlpha

In[6]:= Int(x^2*ln(x)^2) dx from 1 to e

Definite integrals:

More digits | Hide steps

$$\int_1^e x^2 \log^2(x)\, dx = \frac{1}{27}(5 e^3 - 2) \approx 3.6455$$

Possible intermediate steps:

Compute the definite integral:

$$\int_1^e x^2 \log^2(x)\, dx$$

For the integrand $x^2 \log^2(x)$, integrate by parts, $\int f\, dg = f g - \int g\, df$, where

$f = \log^2(x), \quad dg = x^2\, dx,$

$df = \frac{2 \log(x)}{x}\, dx, \quad g = \frac{x^3}{3}$:

$$= \frac{1}{3} x^3 \log^2(x) \Big|_1^e - \frac{1}{3} \int_1^e 2 x^2 \log(x)\, dx$$

Evaluate the antiderivative at the limits and subtract.

$\frac{1}{3} x^3 \log^2(x) \Big|_1^e = \frac{1}{3} e^3 \log^2(e) - \frac{1}{3} 1^3 \log^2(1) = \frac{e^3}{3}$:

$$= \frac{e^3}{3} - \frac{1}{3} \int_1^e 2 x^2 \log(x)\, dx$$

Factor out constants:

$$= \frac{e^3}{3} - \frac{2}{3} \int_1^e x^2 \log(x)\, dx$$

For the integrand $x^2 \log(x)$, integrate by parts, $\int f\, dg = f g - \int g\, df$, where

$f = \log(x), \quad dg = x^2\, dx,$

$df = \frac{1}{x}\, dx, \quad g = \frac{x^3}{3}$:

$$= \frac{e^3}{3} + \left(-\frac{2}{9} x^3 \log(x)\right)\Big|_1^e + \frac{2}{9} \int_1^e x^2\, dx$$

Evaluate the antiderivative at the limits and subtract.

$\left(-\frac{2}{9} x^3 \log(x)\right)\Big|_1^e = \left(-\frac{2}{9} e^3 \log(e)\right) - \left(-\frac{2}{9} 1^3 \log(1)\right) = -\frac{2 e^3}{9}$:

$$= \frac{e^3}{9} + \frac{2}{9} \int_1^e x^2 \, dx$$

Apply the fundamental theorem of calculus.

The antiderivative of x^2 is $\frac{x^3}{3}$:

$$= \frac{e^3}{9} + \frac{2x^3}{27} \Bigg|_1^e$$

Evaluate the antiderivative at the limits and subtract.

$\frac{2x^3}{27} \Big|_1^e = \frac{2e^3}{27} - \frac{2 \times 1^3}{27} = \frac{2}{27}(e^3 - 1)$:

$$= \frac{e^3}{9} + \frac{2}{27}(e^3 - 1)$$

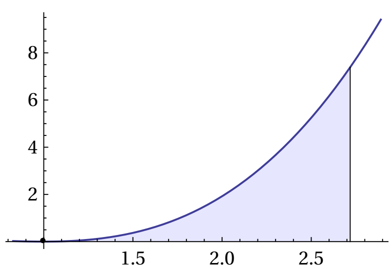
Which is equal to:

Answer:

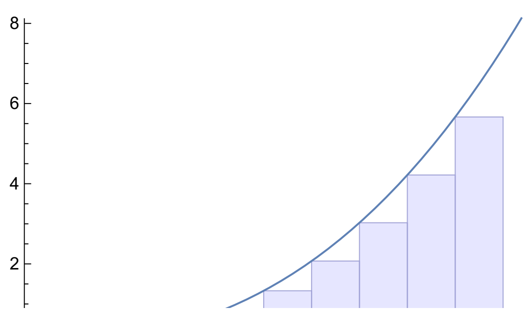
$$= \frac{1}{27}(5e^3 - 2)$$

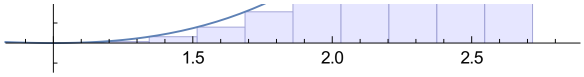
log(x) is the natural logarithm »

Visual representation of the integral:



Riemann sums:





integral: $\frac{1}{27}(5e^3 - 2) \approx 3.64547$

Riemann sum: 3.0374

error: 0.608069

number of subintervals

summation method | left endpoint | midpoint | right endpoint

Indefinite integral:

Step-by-step solution

$$\int x^2 \log^2(x)\, dx = \frac{2x^3}{27} + \frac{1}{3}x^3 \log^2(x) - \frac{2}{9}x^3 \log(x) + \text{constant}$$

WolframAlpha

In[15]:= p'''(q)-p'(q)=0, p(0)=3,p'(0)=-1,p''(0)=1

Assuming "'" is referring to math | Use "''" as a unit instead

Input:

$\{p^{(3)}(q) - p'(q) = 0,\ p(0) = 3,\ p'(0) = -1,\ p''(0) = 1\}$

Autonomous equation:

$-p'(q) + p^{(3)}(q) = 0$

Autonomous equation »

ODE classification:

third- order linear ordinary differential equation

Differential equation solutions:

Approximate form | Hide steps

$p(q) = e^{-q} + 2$

Possible intermediate steps:

Solve $\frac{d^3 p(q)}{dq^3} - \frac{dp(q)}{dq} = 0$, such that $p(0) = 3$, $p'(0) = -1$, and $p''(0) = 1$:

Assume a solution will be proportional to $e^{\lambda q}$ for some constant λ.

Substitute $p(q) = e^{\lambda q}$ into the differential equation:

$$\frac{d^3}{dq^3}(e^{\lambda q}) - \frac{d}{dq}(e^{\lambda q}) = 0$$

Substitute $\frac{d^3}{dq^3}(e^{\lambda q}) = \lambda^3 e^{\lambda q}$ and $\frac{d}{dq}(e^{\lambda q}) = \lambda e^{\lambda q}$:

$\lambda^3 e^{\lambda q} - \lambda e^{\lambda q} = 0$

Factor out $e^{\lambda q}$:

$(\lambda^3 - \lambda) e^{\lambda q} = 0$

Since $e^{\lambda q} \neq 0$ for any finite λ, the zeros must come from the polynomial:

$\lambda^3 - \lambda = 0$

Factor:

$\lambda (\lambda - 1)(\lambda + 1) = 0$

Solve for λ:

$\lambda = -1$ or $\lambda = 0$ or $\lambda = 1$

The root $\lambda = -1$ gives $p_1(q) = c_1\, e^{-q}$ as a solution, where c_1 is an arbitrary constant.
The root $\lambda = 0$ gives $p_2(q) = c_2$ as a solution, where c_2 is an arbitrary constant.
The root $\lambda = 1$ gives $p_3(q) = c_3\, e^{q}$ as a solution, where c_3 is an arbitrary constant.
The general solution is the sum of the above solutions:
$p(q) = p_1(q) + p_2(q) + p_3(q) = c_1\, e^{-q} + c_2 + c_3\, e^{q}$

Solve for the unknown constants using the initial conditions:
Compute $\frac{dp(q)}{dq}$:

$$\frac{dp(q)}{dq} = \frac{d}{dq}(c_1\, e^{-q} + c_2 + c_3\, e^{q})$$
$$= -c_1\, e^{-q} + c_3\, e^{q}$$

Compute $\frac{d^2 p(q)}{dq^2}$:

$$\frac{d^2 p(q)}{dq^2} = \frac{d^2}{dq^2}(c_1\, e^{-q} + c_2 + c_3\, e^{q})$$
$$= c_1\, e^{-q} + c_3\, e^{q}$$

Substitute $p(0) = 3$ into $p(q) = c_1\, e^{-q} + c_3\, e^{q} + c_2$:
$c_1 + c_2 + c_3 = 3$

Substitute $p'(0) = -1$ into $\frac{dp(q)}{dq} = c_3\, e^{q} - c_1\, e^{-q}$:
$-c_1 + c_3 = -1$

Substitute $p''(0) = 1$ into $\frac{d^2 p(q)}{dq^2} = c_1\, e^{-q} + c_3\, e^{q}$:
$c_1 + c_3 = 1$

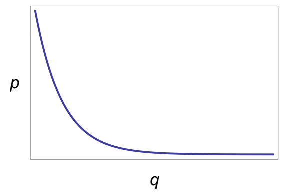
Solve the system:
$c_1 = 1$
$c_2 = 2$
$c_3 = 0$

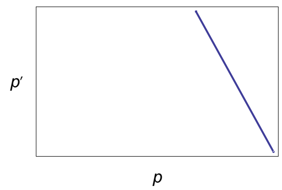
Substitute $c_1 = 1$, $c_2 = 2$, and $c_3 = 0$ into $p(q) = c_1\, e^{-q} + c_3\, e^{q} + c_2$:

Answer:

$$p(q) = e^{-q} + 2$$

Plots of the solution:





WolframAlpha

In[16]:= **y''(x)-3*y'(x)+2*y(x)=sin(x), y(0)=3/10,y'(0)=0**

Input:

$$\left\{y''(x) - 3\,y'(x) + 2\,y(x) = \sin(x),\ y(0) = \frac{3}{10},\ y'(0) = 0\right\}$$

ODE classification:

second- order linear ordinary differential equation

Alternate forms:

$$\left\{y''(x) = 3\,y'(x) - 2\,y(x) + \sin(x),\ y(0) = \frac{3}{10},\ y'(0) = 0\right\}$$

$$\{3\,y'(x) + \sin(x) = y''(x) + 2\,y(x),\ 10\,y(0) = 3,\ y'(0) = 0\}$$

$$\left\{y''(x) - 3\,y'(x) + 2\,y(x) = \frac{1}{2}\,i\,e^{-i\,x} - \frac{1}{2}\,i\,e^{i\,x},\ y(0) = \frac{3}{10},\ y'(0) = 0\right\}$$

Differential equation solutions:

Approximate form

Solve with undetermined coefficients

Hide steps

$$y(x) = \frac{1}{10}\left(e^{x} - e^{2\,x} + \sin(x) + 3\cos(x)\right)$$

Possible intermediate steps:

Solve $-3\,\frac{dy(x)}{dx} + \frac{d^2y(x)}{dx^2} + 2\,y(x) = \sin(x)$, such that $y(0) = \frac{3}{10}$ and $y'(0) = 0$:

The general solution will be the sum of the complementary solution and particular solution.

Find the complementary solution by solving $\frac{d^2y(x)}{dx^2} - 3\,\frac{dy(x)}{dx} + 2\,y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}\left(e^{\lambda x}\right) - 3\,\frac{d}{dx}\left(e^{\lambda x}\right) + 2\,e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2\,e^{\lambda x}$ and $\frac{d}{dx}\left(e^{\lambda x}\right) = \lambda\,e^{\lambda x}$:

$$\lambda^2\,e^{\lambda x} - 3\,\lambda\,e^{\lambda x} + 2\,e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$(\lambda^2 - 3\,\lambda + 2)\,e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:
$\lambda^2 - 3\lambda + 2 = 0$

Factor:
$(\lambda - 2)(\lambda - 1) = 0$

Solve for λ:
$\lambda = 1$ or $\lambda = 2$

The root $\lambda = 1$ gives $y_1(x) = c_1 e^x$ as a solution, where c_1 is an arbitrary constant.
The root $\lambda = 2$ gives $y_2(x) = c_2 e^{2x}$ as a solution, where c_2 is an arbitrary constant.
The general solution is the sum of the above solutions:
$y(x) = y_1(x) + y_2(x) = c_1 e^x + c_2 e^{2x}$

Determine the particular solution to $\frac{d^2 y(x)}{dx^2} + 2\,y(x) - 3\,\frac{dy(x)}{dx} = \sin(x)$ by the method of undetermined coefficients:
The particular solution to $\frac{d^2 y(x)}{dx^2} + 2\,y(x) - 3\,\frac{dy(x)}{dx} = \sin(x)$ is of the form:
$y_p(x) = a_1 \cos(x) + a_2 \sin(x)$

Solve for the unknown constants a_1 and a_2:
Compute $\frac{dy_p(x)}{dx}$:

$$\begin{aligned}\frac{dy_p(x)}{dx} &= \frac{d}{dx}(a_1 \cos(x) + a_2 \sin(x)) \\ &= -a_1 \sin(x) + a_2 \cos(x)\end{aligned}$$

Compute $\frac{d^2 y_p(x)}{dx^2}$:

$$\begin{aligned}\frac{d^2 y_p(x)}{dx^2} &= \frac{d^2}{dx^2}(a_1 \cos(x) + a_2 \sin(x)) \\ &= -a_1 \cos(x) - a_2 \sin(x)\end{aligned}$$

Substitute the particular solution $y_p(x)$ into the differential equation:

$$\frac{d^2 y_p(x)}{dx^2} - 3\,\frac{dy_p(x)}{dx} + 2\,y_p(x) = \sin(x)$$

$$(-a_1 \cos(x) - a_2 \sin(x)) - 3\,(-a_1 \sin(x) + a_2 \cos(x)) + 2\,(a_1 \cos(x) + a_2 \sin(x)) = \sin(x)$$

Simplify:
$(a_1 - 3\,a_2)\cos(x) + (3\,a_1 + a_2)\sin(x) = \sin(x)$

Equate the coefficients of $\cos(x)$ on both sides of the equation:

$$a_1 - 3\,a_2 = 0$$

Equate the coefficients of $\sin(x)$ on both sides of the equation:

$$3\,a_1 + a_2 = 1$$

Solve the system:

$$a_1 = \frac{3}{10}$$
$$a_2 = \frac{1}{10}$$

Substitute a_1 and a_2 into $y_p(x) = a_2 \sin(x) + a_1 \cos(x)$:

$$y_p(x) = \frac{3\cos(x)}{10} + \frac{\sin(x)}{10}$$

The general solution is:

$$y(x) = y_c(x) + y_p(x) = \frac{3\cos(x)}{10} + \frac{\sin(x)}{10} + c_1\,e^x + c_2\,e^{2\,x}$$

Solve for the unknown constants using the initial conditions:

Compute $\frac{dy(x)}{dx}$:

$$\frac{dy(x)}{dx} = \frac{d}{dx}\left(\frac{3\cos(x)}{10} + \frac{\sin(x)}{10} + c_1\,e^x + c_2\,e^{2\,x}\right)$$
$$= \frac{\cos(x)}{10} - \frac{3\sin(x)}{10} + c_1\,e^x + 2\,c_2\,e^{2\,x}$$

Substitute $y(0) = \frac{3}{10}$ into $y(x) = c_1\,e^x + c_2\,e^{2\,x} + \frac{\sin(x)}{10} + \frac{3\cos(x)}{10}$:

$$c_1 + c_2 + \frac{3}{10} = \frac{3}{10}$$

Substitute $y'(0) = 0$ into $\frac{dy(x)}{dx} = c_1\,e^x + 2\,c_2\,e^{2\,x} - \frac{3\sin(x)}{10} + \frac{\cos(x)}{10}$:

$$c_1 + 2\,c_2 + \frac{1}{10} = 0$$

Solve the system:

$$c_1 = \frac{1}{10}$$
$$c_2 = -\frac{1}{10}$$

Substitute $c_1 = \frac{1}{10}$ and $c_2 = -\frac{1}{10}$ into $y(x) = c_1\, e^x + c_2\, e^{2x} + \frac{\sin(x)}{10} + \frac{3\cos(x)}{10}$:

Answer:

$$y(x) = \frac{1}{10}\left(-e^{2x} + e^x + 3\cos(x) + \sin(x)\right)$$

Plots of the solution: